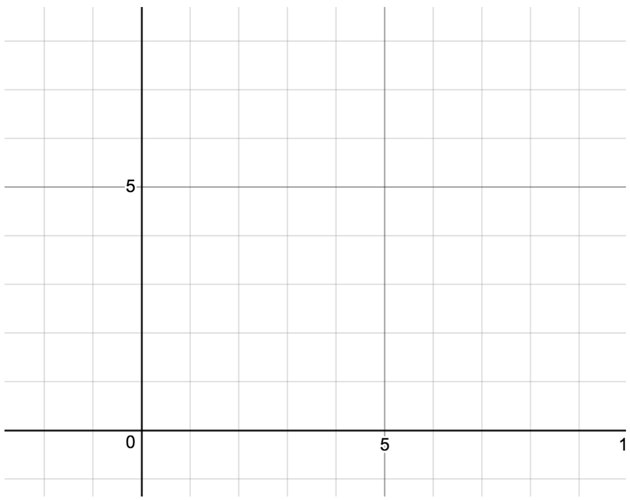
6.1i Review of Inverse Functions

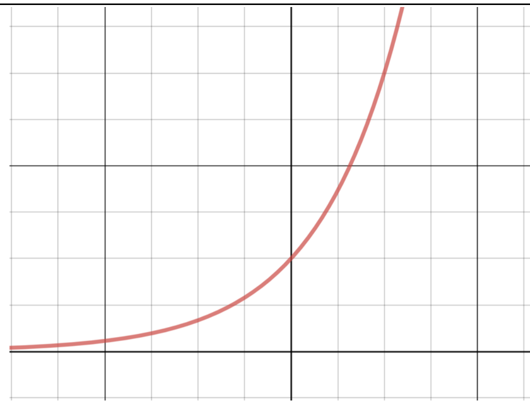
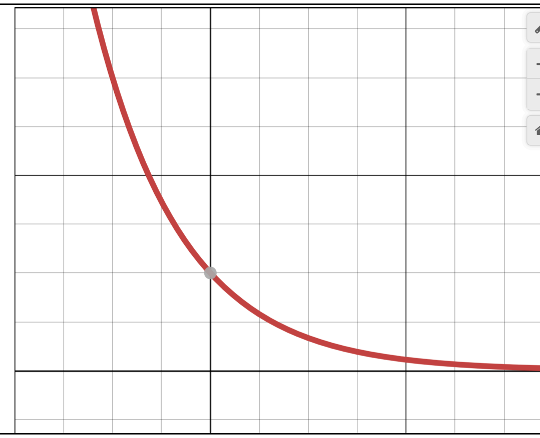
Given $f(x) = \frac{1}{2}x^2 + 2\,;\quad x \geq 0$

- find $f^{-1}(x)$
- find the domain and range of $f(x)$ & $f^{-1}(x)$
- sketch a graph of $f(x)$ & $f^{-1}(x)$
- prove the functions are inverses by showing $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$



6.2i Review of Exponential Functions plus limits (NOTE: not blue pages 6.2*)

Review : Exponential Functions $f(x) = a^x$

	a>1	0<a<1
Graphs		
Domain		
Range		
$\lim\limits_{x\to\infty} a^x$		
$\lim\limits_{x\to-\infty} a^x$		

Formally defining $f(x) = a^x$ for x irrational,

Computing limits: Examples.

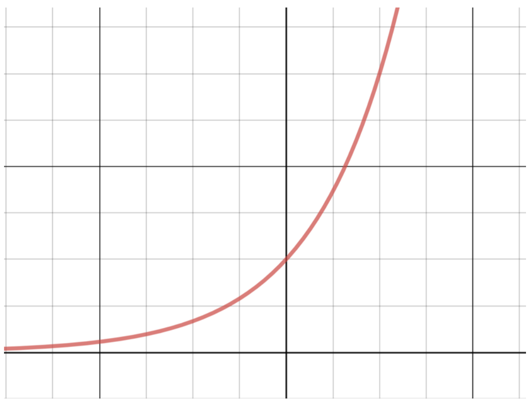
$\lim\limits_{x\to\infty} 3^{-x}$ $\qquad$ $\lim\limits_{x\to 0^+} 5^{1/x}$ $\qquad$ $\lim\limits_{x\to\infty} \frac{2^x}{2^x+1}$

6.3 Review of Logarithmic Functions plus Limits

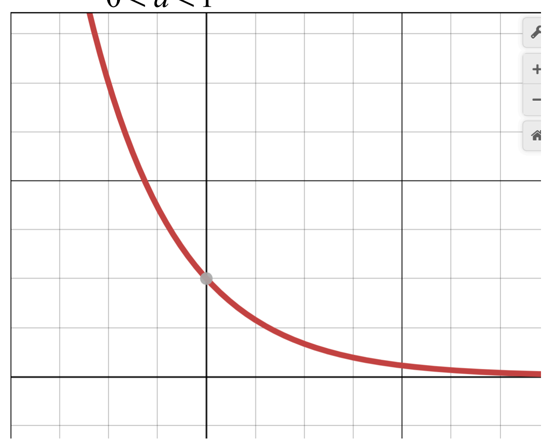
Let's explore the inverse of the exponential function. Based on our knowledge of the relationship of functions to their inverses, we can know a lot about the inverse of $f(x)=a^x$ before we even compute it.

Since we found that for $f(x)=a^x$

$a>1$



$0<a<1$



Domain of $f(x)$ is $(-\infty,\infty)$ $\Rightarrow$ Range of $f^{-1}(x)$ is ________________

Range of $f(x)$ is $(0,\infty)$ $\Rightarrow$ Domain of $f^{-1}(x)$ is ________________

Computing $f^{-1}(x)$ for $f(x)=a^x \Rightarrow y=a^x$. Switching x and y we get $x=a^y$. Now solve for x...?

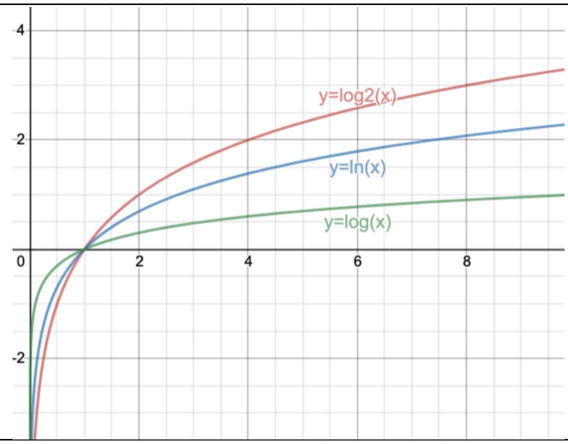
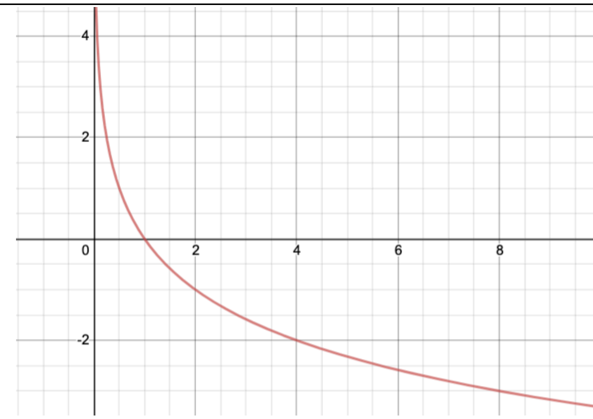
$$\log_a x = y \quad \Leftrightarrow \quad a^y = x$$

The natural logarithm, is a logarithm whose base is e

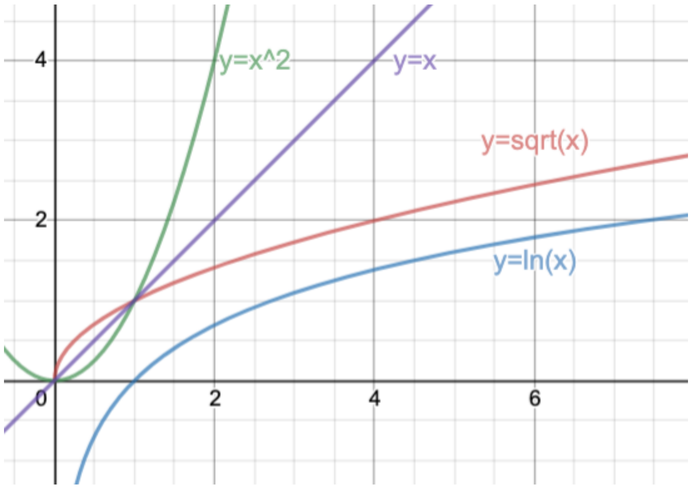
$$\log_e x = \ln x = y \quad \Leftrightarrow \quad e^y = x$$

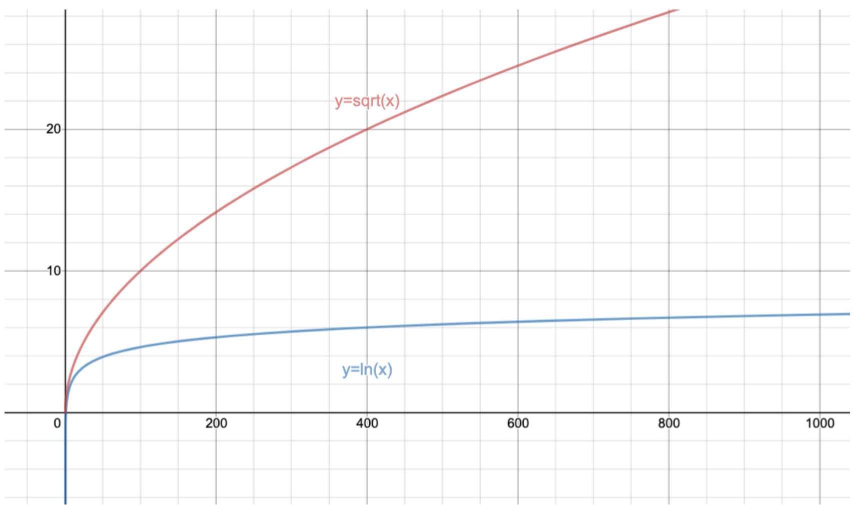
You should be familiar with the basic shape of the logarithm graphs. It will really help when we discuss limits.

$y = \log_a x \quad a > 1$ (includes $\ln x$) $\qquad\qquad y = \log_a x; \quad 0 < a < 1$

<table>
<tr><td></td><td>a>1</td><td>0<a<1</td></tr>
<tr><td>Graphs</td><td>
</td><td></td></tr>
<tr><td>Domain</td><td></td><td></td></tr>
<tr><td>Range</td><td></td><td></td></tr>
<tr><td>$\lim_{x\to\infty} \log_a x =$</td><td></td><td></td></tr>
<tr><td>$\lim_{x\to 0^+} \log_a x =$</td><td></td><td></td></tr>
</table>

<u>The *rate* at which $\ln(x) \to \infty$</u>





Limit Examples:

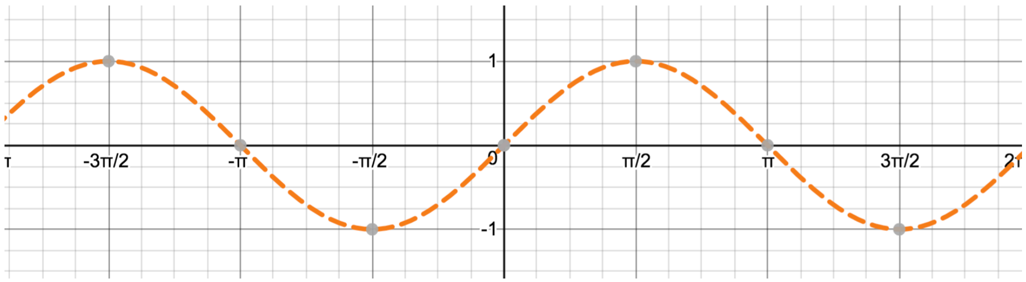
$\lim_{x\to 0^+} \ln(\sin x)$ $\lim_{x\to\infty}\left(\ln(x)-\ln(x+1)\right)$ $\lim_{x\to\infty}\frac{\ln x}{x^2}$

Review LOG PROPERTIES, solving log and exponential equations and graphing log and exponential functions.

6.6i Review of Inverse Trigonometric Functions

Inverse Sine Function

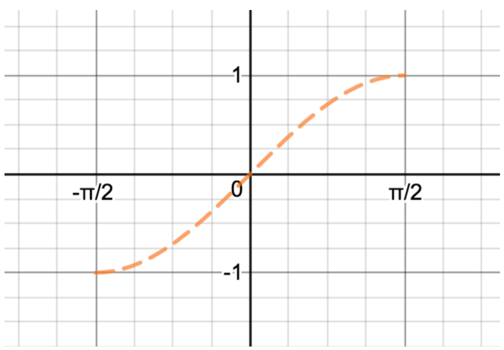
Does $g(x) = \sin(x)$ have an inverse? ______________________________



What restriction would we need to make so that at least a piece of this function has an inverse?

Given $f(x) = \sin(x);$ ______________

1) Find $f^{-1}(x)$
2) Graph $f(x)$ and $f^{-1}(x)$.
3) Find the domain and range of $f(x)$ and $f^{-1}(x)$.



We define $y = \sin^{-1}(x)$ or $y = \arcsin(x)$ to mean $\begin{cases} \sin(y) = x \\ AND \\ -\frac{\pi}{2} \le y \le \frac{\pi}{2} \end{cases}$

Note: Both the input and output of this function are real numbers, but it is sometimes helpful to think in terms of angles.

that is let $\theta = \sin^{-1}(x)$ or $\theta = \arcsin(x)$ mean $\begin{cases} \sin(\theta) = x \\ AND \\ -\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \end{cases}$

For example:

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$ $\qquad$ $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\sin(angle) = number$ $\qquad$ $\sin^{-1}(number) = angle\ in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Finding exact values of the inverse sine function for special inputs: (like:______________________)

Ex: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

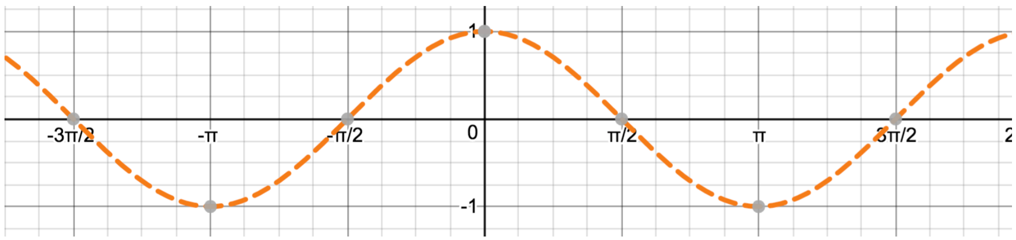
Set $\theta = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ and re-write according to the definition as ___________________

In words: $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ *is the real number (or angle) in* $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ *whose sine (or y value on the unit circle) is* $\frac{\sqrt{3}}{2}$

Ex: $\sin^{-1}\left(\frac{-\sqrt{2}}{2}\right)$

Since $y = \sin^{-1}(x)$ is a *function*, ______________________________

Inverse Cosine Function



What restriction would we need to make so that at least a piece of this function has an inverse?

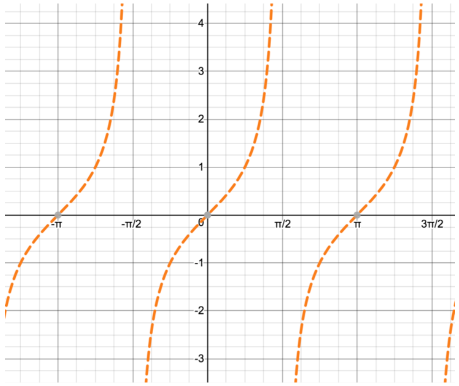
of angles. The development is similar to $\sin^{-1}(x)$, review as needed.

let $\theta = \cos^{-1}(x)$ or $\theta = \arccos(x)$ mean $\begin{cases} \cos(\theta) = x \\ AND \\ 0 \le \theta \le \pi \end{cases}$

Finding exact values of the inverse cosine function for special inputs:

Ex: $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

Inverse Tangent Function



What restriction would we need to make so that at least a piece of this function has an inverse?
Again, the development is similar to $\sin^{-1}(x)$, review as needed.

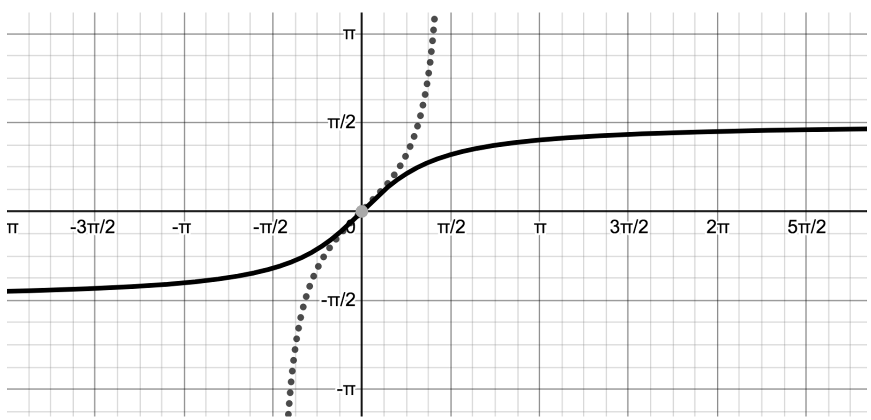
Finding exact values of the inverse tangent function for special inputs:

Ex: $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$ =_______________ $\tan^{-1}(-1)$=_______________

Of note about the inverse tangent function,

$\lim_{x\to\infty} \tan^{-1}(x)$= _______________ and $\lim_{x\to-\infty} \tan^{-1}(x)$= _______________

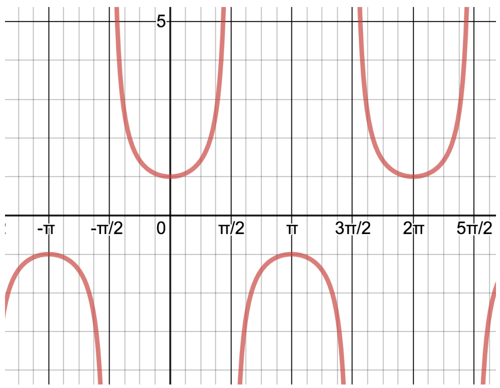
as can be seen in the graph.



The other inverse trig. functions

The other inverses:

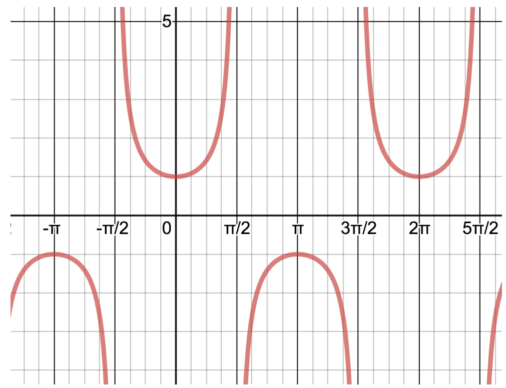
$f(x) = \sec(x)$



"Trig friendly" restrictions: ______________

$\sec^{-1}(2) =$ _________________

$\sec^{-1}(-\sqrt{2}) =$ _________________



"Calculus friendly" restrictions:________________

$\sec^{-1}(2) =$ _________________

$\sec^{-1}(-\sqrt{2}) =$ _________________

See the book for $\csc^{-1}(x)$ and $\cot^{-1}(x)$. You do not need to memorize these restrictions, but do know how to find values for a given set of restrictions.

<u>Mixed Compositions –</u>

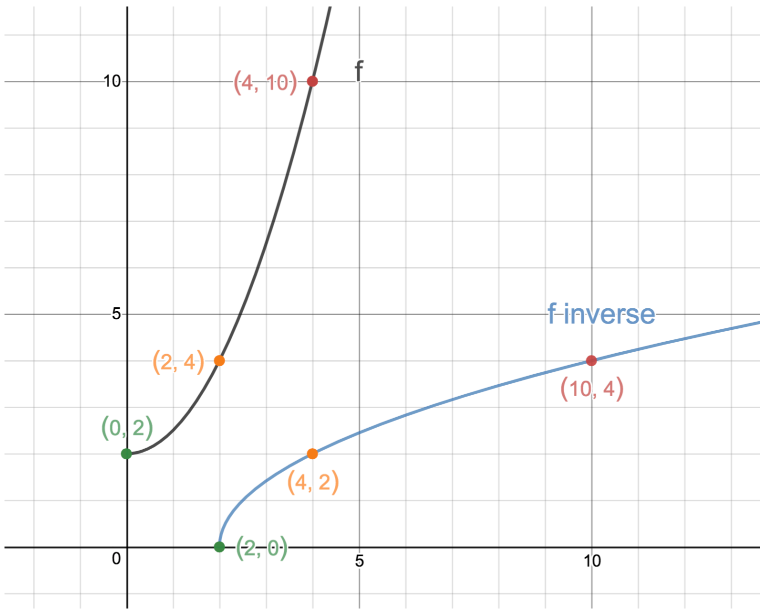
Find exact values:

$\sin\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$ $\tan\left(\sin^{-1}\left(\frac{-2\sqrt{5}}{5}\right)\right)$ $\cos\left(\tan^{-1}x\right)$

6.1ii Derivatives of Inverse Functions

Now for some Calculus. Let's look at some derivatives from the earlier example $f(x)=\frac{1}{2}x^2+2$; $x\ge 0$.

https://www.desmos.com/calculator/znclra2xmj



$f'=$ $\qquad$ $\left(f^{-1}\right)'=$

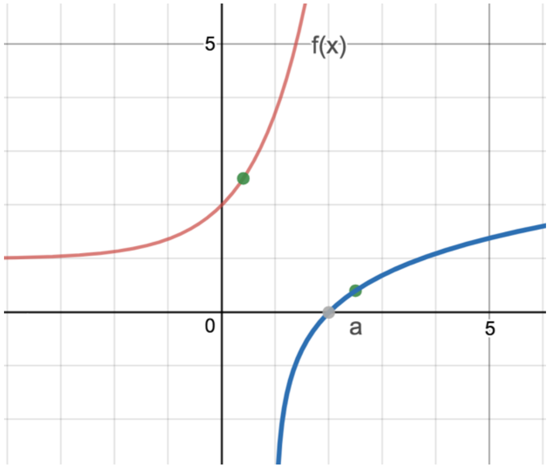
(x,y)	$f'(x)$	(x,y)	$\left(f^{-1}\right)'(x)$

For every point (x,y) on the graph of f there is a point (y,x) on the graph of f^{-1}. I will call these companion points.

What is the relationship between the derivative of f f at point P and the derivative of f^{-1} at P's companion point?

Label the points shown:

$$\frac{d}{dx}\left(f^{-1}\right)'(a) =$$



How do we find $f^{-1}(a)$ if we don't have the formula for $f^{-1}(x)$?

Example: For $f(x) = 2x + \cos x$, find $\left(f^{-1}\right)'(1)$ or $\frac{d}{dx}\left[f^{-1}(x)\right]_{x=1}$

Alternate approach: ______________________________

6.2ii: Derivatives of Exponential Functions:

Goal: find a formula for the derivative of $f(x) = a^x$

$$\frac{d}{dx}\left[a^x\right] = f'(x\ ) = \lim_{h\to 0}\frac{f(x+h)-f(x\ )}{h} =$$

Suppose we just try to find the derivative at a specific point, x=0.

Recall $f'(x_0) = \lim_{h\to 0}\frac{f(x_0+h)-f(x_0)}{h}$, so $f'(0)$=

Then so far, $\frac{d}{dx}\left[a^x\right] = a^x \lim_{h\to 0}\frac{a^h-1}{h} = a^x f'(0)$

Lets look further at this limit. Approximating $\lim_{h\to 0}\frac{a^h-1}{h}$ numerically for a=2 and a=3:

h	$\frac{2^h-1}{h}$
1	1
.5	0.82842712
.1	0.71773463
.01	0.69555501
.001	0.69338746
.0000001	0.6931472

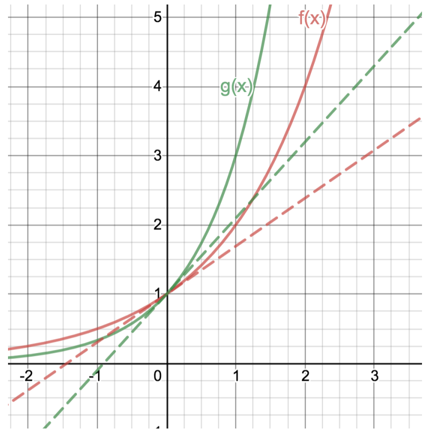
h	$\frac{3^h-1}{h}$
1	2
.5	1.4641016
.1	1.1612317
.01	1.1046692
.001	1.099216
.0000001	1.0986123

Limit exist? Need more values?

So for $f(x)=2^x$, $f'(0)=\lim_{h\to 0}\frac{2^h-1}{h}$ ≈ 0.693 and similarly, for $g(x)=3^x$, $g'(0)=\lim_{h\to 0}\frac{3^h-1}{h}$ ≈1.099. (See graph)

Then the general derivative, $\frac{d}{dx}\left[2^x\right]\approx 2^x(0.693)$ and $\frac{d}{dx}\left[3^x\right]\approx 3^x(1.099)$

We found above, for x=0 with $f(x)=2^x$ and $g(x)=3^x$



So far, we are finding it difficult to find $f'(0)$ and thus $f'(x)$ exactly, at least for a =2, 3. It it possible there are some values for "a" for which these computations provide and exact value for $f'(0)$. What if we work backwards?

Looking at the above graph, there must be some choice of "a" between 2 and 3, such that the slope of the tangent is exactly ONE. Let's call that choice of "a", "b". We don't know the value of "b", but we are choosing "b" such that $\lim_{h\to 0}\frac{b^h-1}{h}=1$, so that if $f(x)=b^x$, then $f'(0)=1$ and in this *special* case, the general derivative is $\frac{d}{dx}\left[b^x\right]=b^x f'(0)=b^x$.

Note: The book uses the letter "e" instead of "b" here, giving some indication that this special number is the natural exponential, but at this point we don't <u>know</u> that. All we know is that "e" is a special number between 2 and 3, such that for $\lim_{h\to 0}\frac{e^h-1}{h}=1$. In this *special* case, $\frac{d}{dx}\left[e^x\right]=e^x$.

<u>Derivative Examples Find the following derivatives:</u>

$y=x^2e^x$

$y=\frac{3\cos x}{e^x}$

Chain Rule formula:

If $f(x)=\sin(x)$, then $f(x^2)=$ ________________ is a composite function requiring the chain rule to differentiate.

Similarly, if If $f(x)=e^x$, $f(x^2)=$_____________ is a composite function requiring the chain rule to differentiate.

Note: The “inner function” is the exponent.

The book writes a separate formula for this situation, $\frac{d}{dx}[e^u]=e^u\frac{du}{dx}$ but it is really not necessary if you understand the chain rule.

Examples:

Integral Formula:

Every differentiation formula provides an integral formula in reverse. $\int e^x \, dx = e^x + C$

Examples:

$\int \sin(3x)\, dx$

$\int e^{3x} \, dx$

$\int x^2 e^{x^3} \, dx$

$\int \frac{e^{1/x}}{x^2} \, dx$

$\int_2^3 e^{2-x} \, dx$

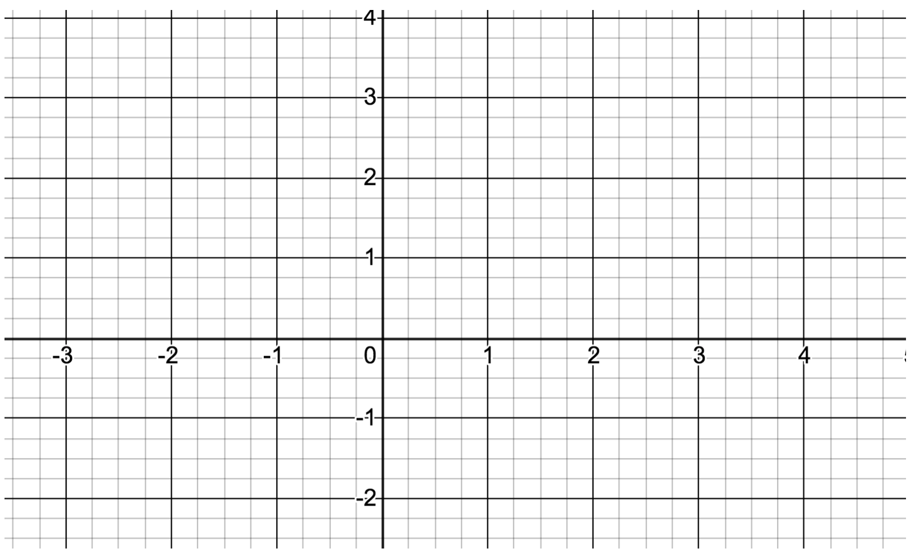
(Watch notation of limits if you don't change to u's limits)

See Estimating e, page 297.

6.2ii continued Application Examples:

Find absolute extrema of $f(x) = x^2 e^{-x/2}$ on [-1,6] See “Closed Interval Method”

Graph $f(x) = e^{1/x}$.



6.4 Derivatives of Logarithmic Functions

Derivation of $\frac{d}{dx}\left[\ln(x)\right]$:

Let $f(x) = e^x$; $f^{-1}(x) =$ ________________-

Examples: Find $f'(x)$

$f(x) = x^2 \ln x$

$f(x) = \ln(\sin x)$

$f(x) = \ln x^3$

$f(x) = \left(\ln x\right)^3$

$f(x) = \ln\left(\frac{\sqrt{x^2+1}}{x^3}\right)$

$f(x) = \ln|x|$

This leads us to a very important result:

$$\int$$

Examples:

$$\int \frac{3x}{x^2+4}\,dx \qquad \int \frac{\ln t}{t}\,dt \qquad \int \tan(x)\,dx$$

So we found $\frac{d}{dx}\left[e^x\right]=e^x$ and $\frac{d}{dx}\left[\ln x\right]=\frac{1}{x}$

But what about the general exponential and logarithmic functions?

We started by trying to develop a formula for $\frac{d}{dx}\left[a^x\right]$

To work with a^x we need only remember that by using the properties of logarithms a^x can be written in terms of e as:

$$\boxed{a^x = e^{x\ln a}}$$ (Why?)

Then $\frac{d}{dx}\left[a^x\right] = \frac{d}{dx}\left[e^{x\ln a}\right] =$ ______________________________

Example:

$\frac{d}{dx}\left[3^x\right] =$

$\frac{d}{dx}\left[5^{x^2}\right] =$

$\int 4^x \, dx$

As for the general logarithm, $\log_a x$, we can use the ___________________________ formula to write $\log_a x$ in terms of $\ln x$.

$$\boxed{\log_a x = \frac{\ln x}{\ln a}}$$

Then

$$\frac{d}{dx}\left[\log_a x\right] = \frac{d}{dx}\left[\frac{\ln x}{\ln a}\right] = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Find $\dfrac{dy}{dx}$: $y = \log_8(x)$ $y = \log_7(2x)$

Logarithmic Differentiation:

A method of using _________________________ to differentiate ____ and

functions involving exponential expressions with___________________ base and exponent.

Example: Find y' if $y = \dfrac{x^3(2x+5)^4}{\sqrt{x^2-8}}$. Quotient, product, chain rule……..

Example: $\dfrac{d}{dx}\left[x^x\right]$

See in book, e as a limit. $e=\lim_{x\to 0}(1+x)^{1/x}=\lim_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x$

6.6ii Derivatives of Inverse Trigonometric Functions

Develop formula for the derivative of $f(x) = \sin^{-1}(x)$

Similarly, we can derive the formulas for the other inverse trig function derivatives.

$\frac{d}{dx}\left[\sin^{-1}(x)\right] =$ ______________________ $\frac{d}{dx}\left[\csc^{-1}(x)\right] =$ ______________________

$\frac{d}{dx}\left[\cos^{-1}(x)\right] =$ ______________________ $\frac{d}{dx}\left[\sec^{-1}(x)\right] =$ ______________________

$\frac{d}{dx}\left[\tan^{-1}(x)\right] =$ ______________________ $\frac{d}{dx}\left[\cot^{-1}(x)\right] =$ ______________________

Examples:

Integration. From the above derivative formulas, we gain the following antiderivative formulas:

$\int \frac{1}{\sqrt{1-x^2}}\,dx =$ ____________________ $\int \frac{1}{1+x^2}\,dx =$ ____________________ $\int \frac{1}{x\sqrt{x^2-1}}\,dx =$ ____________________

Examples:

$$\int_0^{1/2} \frac{3}{\sqrt{1-x^2}}\,dx$$

$$\int \frac{1}{x\sqrt{9x^2-1}}\,dx$$

$$\int \frac{1}{4+x^2}\,dx$$

Generalizing Formulas (a>0):

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right)+C \qquad \int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C \qquad \int \frac{1}{x\sqrt{x^2-a^2}}\,dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+C$$

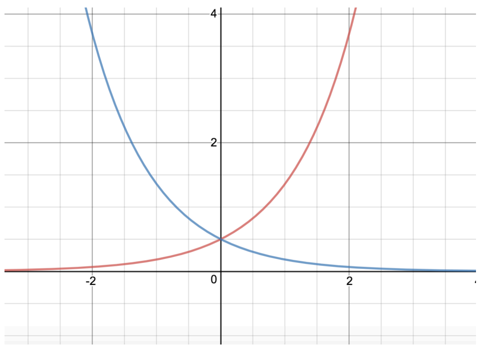
$$\int_{\ln(2)}^{\ln(2/\sqrt{3})} \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\,dx$$

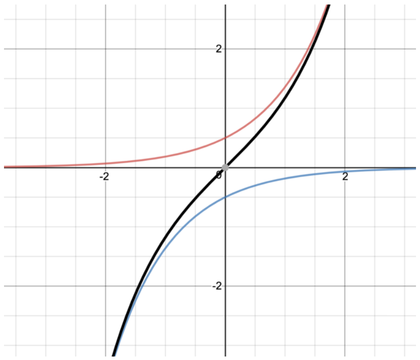
6.7 Hyperbolic Functions- Quick Introduction

<u>Hyberbolic Cosine</u>: $\cosh(x)=\dfrac{e^x+e^{-x}}{2}$ <u>Hyperbolic Sine</u>: $\sinh(x)=\dfrac{e^x-e^{-x}}{2}$

You need to memorize the above formulas, but those are the only formulas I require in this section.

<u>Graphs:</u> (Graphical Addition)





<u>Other Hyperbolics:</u>

$$\tanh x=\frac{\sinh x}{\cosh x} \qquad \coth x=\frac{\cosh x}{\sinh x}=\frac{1}{\tanh x}$$
$$\operatorname{sech} x=\frac{1}{\cosh x} \qquad \operatorname{csch} x=\frac{1}{\sinh x}$$

Identities:

Similar, but different identities to the "Circular" Trigonometric Functions

EX: $\cosh^2(x)-\sinh^2(x)=1$

See book for more. You do not need to memorize, but be able to derive.

Derivatives:

Derivation $\frac{d}{dx}\left[\sinh(x)\right]$

Similarly we can find:

$$\frac{d}{dx}(\sinh x) = \cosh x \qquad \frac{d}{dx}(\cosh x) = \sinh x$$
$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x \qquad \frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$$
$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x \qquad \frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

Integrals:

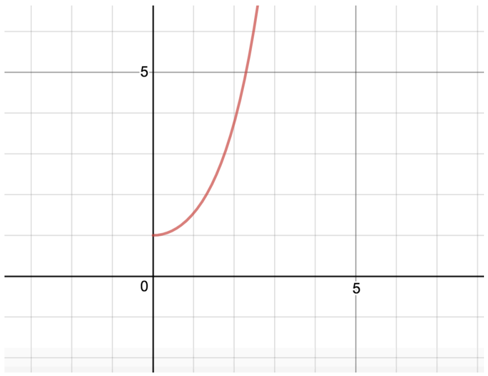
$$\int \sinh x \, dx = \cosh x + c$$
$$\int \cosh x \, dx = \sinh x + c$$
$$\int \tanh x \, dx = \ln(\cosh x) + c$$
$$\int \coth x \, dx = \ln|\sinh x| + c$$
$$\int \operatorname{sech} x \, dx = \tan^{-1}|\sinh x| + c$$
$$\int \operatorname{csch} x \, dx = \ln\left|\tanh \frac{x}{2}\right| + c$$

See book's examples.

Inverse Hyperbolics

(See book's derivation of $\sinh^{-1}(x)$)

Given $f(x) = \cosh(x); x \geq 0$, find $f^{-1}(x)$



<u>Similarly, can obtain:</u>

$\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$

$\coth^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x} \quad (|x| > 1)$

$\cosh^{-1} x = \ln(x + \sqrt{x^2 - 1}) \quad (x \geq 1)$

$\operatorname{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right) \quad (0 < x \leq 1)$

(x>1)

$\tanh^{-1} x = \frac{1}{2}\ln\frac{1+x}{1-x} \quad (|x| < 1)$

$\operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right) \quad (x \neq 0)$

Derivatives of Inverse Hyperbolics

$$\frac{d}{dx}\left[\cosh^{-1}(x)\right]$$

Similarly, can obtain:

$$\frac{d}{dx}\sinh^{-1}x = \frac{1}{\sqrt{1+x^2}}$$

$$\frac{d}{dx}\cosh^{-1}x = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}\tanh^{-1}x = \frac{1}{1-x^2} \quad (|x|<1)$$

$$\frac{d}{dx}\coth^{-1}x = \frac{1}{1-x^2} \quad (|x|>1)$$

$$\frac{d}{dx}\operatorname{sech}^{-1}x = \frac{-1}{x\sqrt{1-x^2}} \quad (0<x<1)$$

$$\frac{d}{dx}\operatorname{csch}^{-1}x = \frac{-1}{|x|\sqrt{1+x^2}} \quad (x\neq 0)$$

Which lead to our ultimate goal in studying these functions in *this* course….

Integration Formulas

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\,dx = \cosh^{-1} x + C \quad (x>1)$$

$$\int \frac{1}{1-x^2}\,dx = \begin{Bmatrix} \tanh^{-1} x + C & |x| < 1 \\ \coth^{-1} x + C & |x| > 1 \end{Bmatrix} = \frac{1}{2}\ln\left|\frac{1+x}{1-x}\right| + C$$

$$\int \frac{1}{x\sqrt{1-x^2}}\,dx = -\operatorname{sech}^{-1}|x| + C$$

$$\int \frac{1}{x\sqrt{1+x^2}}\,dx = -\operatorname{csch}^{-1}|x| + C$$

Examples:

$$\int_0^1 \frac{1}{\sqrt{1+x^2}}\,dx$$

$$\int \frac{dx}{\sqrt{9x^2 - 25}}$$

$$\int \frac{e^{2x}}{\sqrt{1 - e^{4x}}}\,dx$$

6.8: L'Hospital's Rule

Limit review problems:

$$\lim_{x\to 2}\frac{x^2-4}{x-2} \qquad \lim_{x\to\infty}\frac{x^5}{3x^5+x+4} \qquad \lim_{x\to 0}\frac{\sin x}{x} \qquad \lim_{x\to 3^+}\frac{7}{x-3}$$

Important note on notation: Infinity is not a number, don't treat it like one!

Now consider:

$$\lim_{x\to\infty}\frac{e^x}{x} \qquad \lim_{x\to\infty}\frac{\ln x}{x} \qquad \lim_{x\to 0}\frac{1-\cos x}{x^2}$$

Indeterminate Forms of Type ________________ or ___________________

<u>L'Hospital's Rule</u>: Suppose that f and g are differentiable and that $g'(x) \neq 0$ on an open interval I that contains a (except possible at a), if the $\lim_{x\to a}\frac{f(x)}{g(x)}$ is of the indeterminate type $\frac{0}{0}$ or $\frac{\pm\infty}{\pm\infty}$, then $\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$ if the limit on the right exists (or is infinite). This is also valid for one sided limits or limits as $x \to \pm\infty$.

See proof for $\frac{0}{0}$ case in book. Case $\frac{\pm\infty}{\pm\infty}$ is in Advanced Calculus

<u>Geometric idea:</u>

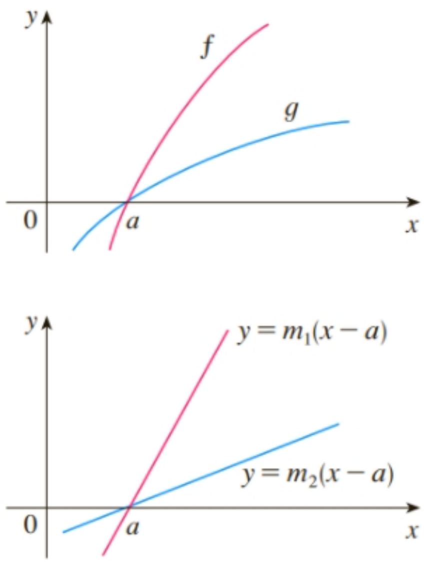


FIGURE 1

Figure 1 suggests visually why l'Hospital's Rule might be true. The first graph shows two differentiable functions f and g, each of which approaches 0 as $x \to a$. If we were to zoom in toward the point $(a, 0)$, the graphs would start to look almost linear. But if the functions actually *were* linear, as in the second graph, then their ratio would be

$$\frac{m_1(x-a)}{m_2(x-a)} = \frac{m_1}{m_2}$$

which is the ratio of their derivatives. This suggests that

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

Examples:

$$\lim_{x\to\infty}\frac{e^x}{x} \qquad \lim_{x\to\infty}\frac{\ln x}{x} \qquad \lim_{x\to 0}\frac{1-\cos x}{x^2}$$

Sometimes, the previous methods learned are preferable: $\lim_{x\to\infty}\frac{x^5}{3x^5+x+4}$

If L'Hospital's rule is applied when it should not be, it may yield the wrong answer: $\lim_{x\to 3^+}\frac{7}{x-3}$

Other Indeterminate Forms: $0\bullet\pm\infty,\ \infty-\infty,\ \infty^0,\ 0^0, 1^\infty$

$$0\bullet\pm\infty$$

If the $\lim_{x\to a} f(x)g(x)$ is of the indeterminate type $0\bullet\pm\infty$, then rewrite fg as ___________ or _____________ which will then cause the indeterminate form ______ or. ________ , allowing L'Hospital's rule to be used.

Examples:

$\lim_{x\to 0^+} x^2 \ln x$ $\lim_{x\to -\infty} xe^x$

$\infty - \infty$

If the $\lim_{x\to a}(f(x)-g(x))$ is of the indeterminate type $\infty-\infty$,then try to algebraically manipulate to the indeterminate form _______ or. _________ , allowing L'Hospital's rule to be used.

Examples:

$\lim_{x\to 0^+}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$ $\lim_{x\to\infty}(x^2-x)$ $\lim_{x\to\infty}(x^2+x)$

Note: ___________________

$$\boxed{\infty^0,\ 0^0,\ 1^\infty}$$

If the $\lim_{x\to a} f(x)^{g(x)}$ is of the indeterminate type $\infty^0, 0^0, 1^\infty$, use logarithms/exponentials to rewrite $f(x)^{g(x)}$ in the form $e^{g(x)\ln(f(x))}$. Now the exponent $g(x)\ln(f(x))$ is of the form $0 \bullet \pm\infty$ and we can use above methods. (Alternately, let $y = f(x)^{g(x)}$, take ln of both sides, then take the limit and solve for y)

Examples:

$$\lim_{x\to 0^+} x^{\sin x}$$

$$\lim_{x\to\infty}\left(\frac{1}{x}\right)^x$$

Note: ________________